

A STRUCTURED CONSTITUTIVE MODEL FOR SIMULATING THE BEHAVIOUR OF AN OVERCONSOLIDATED BONDED CLAY

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Abstract. The paper presents some improvements in the formulation of a kinematic hardening constitutive soil model incorporating structure initially proposed for soft clays. For the modelling of overconsolidated bonded clay the elastic formulation was deemed more important. Two different alternatives, one purely empirically based the other with a background in thermodynamics were implemented. It was also found that a smooth elasto-plastic transition was required to avoid a spurious stiffness degradation response. Consequently, the hardening modulus formulation of the model was modified. The paper presents some results from a parametric analysis of the triaxial drained response of a material tailored to mimic London clay. The results chosen do not show a major difference between the chosen alternative elastic formulations, although both do improve the original model response. On the other hand the importance of ensuring a smooth elasto-plastic transition is clearly highlighted.

1 INTRODUCTION

Critical state soil mechanics led to a significant improvement of predictions of soil behaviour by introducing specific volume as an additional state variable. The essential features of the classical critical state models are that on a primary loading large plastic strains occur, but on subsequent unload - reload cycles within the yield surface only elastic strains are predicted. Later research on the behaviour of soil in the small strain and very small strain range [1] revealed that the assumption of elastic behaviour inside the state boundary surface is not acceptable due to the non-linearity of soil behaviour in the small strain range. Models based on the concepts of kinematic hardening [2] and bounding surface [3] plasticity seem to provide and improvement, over simple elasto-plastic constitutive models, in modelling the

highly nonlinear and inelastic behaviour of soils. These models allows for plasticity and nonlinearity to be invoked within the conventionally defined yield surface. A kinematic hardening extension of a Cam clay-like model was proposed by [4]. Later, [5] extended this model to simulate the behaviour of natural clays damaged only by plastic straining.

This chapter describes a kinematic hardening soil model for structured soils, its numerical implementation and validation. The kinematic hardening constitutive model (KHSM) used is based on [5] here modified to improve the model response in the small strain region and to predict a smooth variation in stiffness. The chapter starts with a brief description of the KHSM formulated in the general stress space. Modifications to the original model are presented in section 3. Section 4 presents some parametric studies of the performance of the model for the simulation of drained triaxial test on London clay.

2 MODEL FORMULATION

3.1 Original KHSM

The model contains three surfaces in stress space: a kinematic yield surface (f_b), a structure surface (F) and a reference surface (f_r) as shown in Figure 1. The bubble surface separates the elastic response from the elasto-plastic response while the structure surface position defines the current structure magnitude and anisotropy of the structure. The size of the structure surface reduces, due to plastic strain, towards the reference surface which defines the behaviour of the non-structured or remoulded material.

$$\begin{aligned}
 f_r &= \frac{3}{2M_\theta^2} s : s + (p' - P_c)^2 - (P_c)^2 = 0 \\
 f_b &= \frac{3}{2M_\theta^2} (s - s_\alpha) : (s - s_\alpha) + (p' - p_\alpha)^2 - (R P_c)^2 = 0 \\
 F &= \frac{3}{2M_\theta^2} (s) : (s) + (p' - r P_c)^2 - (r P_c)^2 = 0
 \end{aligned} \tag{1}$$

where p and s are the mean pressure and the deviatoric stress tensor, $\{p'_\alpha \mathbf{I}, s_\alpha\}^T = \bar{\alpha}$ denotes the location of the centre of the bubble and $\{r P_c \mathbf{I}\}^T = \hat{\alpha}$ denotes the centre of the structure surface. R is a model parameter and M_θ the slope of the critical state line. This is a function of the Lode angle θ following a proposal in [6]

$$M_\theta = M \left(\frac{2\alpha^4}{1 + \alpha^4 + (1 - \alpha^4) \sin(3\theta)} \right)^{1/4} \tag{2}$$

where M is the slope of the CSL under triaxial compression ($\theta = -30^\circ$).

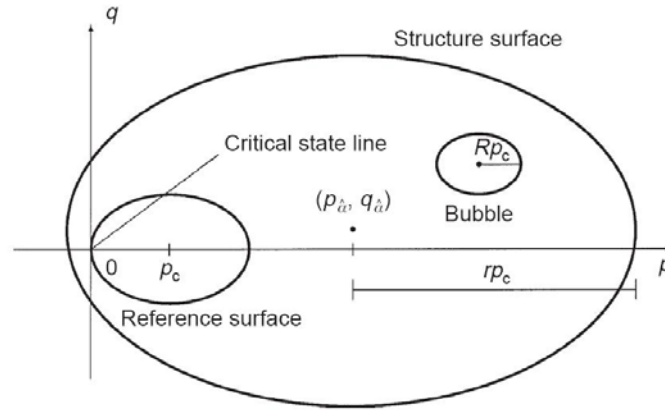


Figure 1: KHSM constitutive model [5]

The scalar variable r , is assumed to be a monotonically decreasing function of the plastic strains and represents the progressive degradation of the material. The incremental form of the destructuration law is written as,

$$\begin{aligned} \dot{r} &= -\frac{k}{(\lambda^* - \kappa^*)} (r-1) \dot{e}_d \\ \dot{e}_d &= \left[(1-A)(\dot{\varepsilon}_v^p)^2 + A(\dot{\varepsilon}_q^p)^2 \right]^{1/2} \end{aligned} \quad (3)$$

A is a non-dimensional scaling parameter, $\dot{\varepsilon}_v^p$ is the plastic volumetric strain rate, $\dot{\varepsilon}_q^p$ is the equivalent plastic shear strain rate and \dot{e}_d is the destructuration strain rate.

In line with Cam-clay, a volumetric hardening rule is adopted, whereby the change in size of the reference surface, P_c , is controlled only by plastic volumetric strain rate, $\dot{\varepsilon}_v^p$,

$$\frac{\dot{P}_c}{P_c} = \frac{\dot{\varepsilon}_v^p}{\lambda^* - \kappa^*} \quad (4)$$

It can be shown that the plastic multiplier, $\dot{\gamma}$, can be computed as:

$$\dot{\gamma} = \frac{1}{H} (\mathbf{n} : \dot{\boldsymbol{\sigma}}) = \frac{1}{H_c} (\mathbf{n} : \dot{\boldsymbol{\sigma}}_c) \quad (5)$$

where the plastic scalar moduli H and H_c are functions of state associated with $\dot{\boldsymbol{\sigma}}$ and $\dot{\boldsymbol{\sigma}}_c$, respectively. $\boldsymbol{\sigma}_c$ is the conjugate stress tensor, defined as the point on the structure surface having the same outward normal as the current stress point $\boldsymbol{\sigma}$ on the bubble. The conjugate hardening modulus H_c is derived from the consistency condition on the structure surface for the case where the bubble and the structure surface are in contact. The explicit expression is,

$$H_c = \frac{2rP_c}{\lambda^* - \kappa^*} \left[\left(2(p - p_a) + \frac{k(1-r)}{r} \frac{\dot{e}_d}{\dot{\gamma}} \right) ((p - p_a) + RP_c) \right] \quad (6)$$

The variation of the hardening modulus within the structure surface is described by an interpolation rule along the distance b , which connects the current stress state on the yield surface with its conjugate point on the structure surface. Hence,

$$H = H_c + \frac{BP_c^3}{\lambda^* - \kappa^*} \left(\frac{b}{b_{\max}} \right)^\Psi \quad (7)$$

Where Ψ and B are model parameters, b is the distance between current stress and conjugate stress, and b_{\max} its maximum as defined below

$$b = \mathbf{n} : (\boldsymbol{\sigma}'_c - \boldsymbol{\sigma}') \quad (8)$$

$$b_{\max} = 2 \left(\frac{r}{R} - 1 \right) \mathbf{n} : (\boldsymbol{\sigma}' - \bar{\boldsymbol{\alpha}})$$

If a stress increment requires movement of the bubble relative to the structure surface, a geometric kinematic hardening rule is invoked to describe this movement. The translation rule of the centre of the bubble $\bar{\boldsymbol{\alpha}}$ is,

$$\dot{\bar{\boldsymbol{\alpha}}} = \bar{\boldsymbol{\alpha}} \left(\frac{\dot{P}_c}{P_c} + \frac{\dot{r}}{r} \right) + \frac{\dot{\gamma}H - \mathbf{n} : \left[\frac{\dot{P}_c}{P_c} \boldsymbol{\sigma}' + \frac{\dot{r}}{r} \bar{\boldsymbol{\alpha}} \right]}{\mathbf{n} : (\boldsymbol{\sigma}'_c - \boldsymbol{\sigma}')} \left[\frac{\boldsymbol{\sigma}' - \bar{\boldsymbol{\alpha}}}{R} - \frac{\boldsymbol{\sigma}' - \hat{\boldsymbol{\alpha}}}{r} \right] \quad (9)$$

3.2 Modified elastic behaviour

The original elastic formulation in KHSM was

$$K = \frac{p'}{\kappa^*} \quad (10a)$$

$$G = \frac{3p'(1+2\nu)}{2\kappa^*(1+\nu)} \quad (10b)$$

As an alternative to the equation (8b) the shear modulus can be described by an empirically based equation proposed in [7]

$$\frac{G}{p_r} = A_g \left(\frac{p'}{p_r} \right)^{n_g} R_o^{m_g} \quad (11)$$

Where A_g , n_g and m_g are dimensionless parameters p_r , is a reference pressure (1 kPa) and

$R_o = 2P_c/p'$, is the isotropic overconsolidation. Correlations of all the parameters entering the equation with plasticity index are given in [7].

According to [8] simplistic models in which tangent moduli are arbitrarily defined as functions of stress can lead to a non-conservative response, in violation of the laws of thermodynamics. In contrast, an hyper-elastic approach guarantees thermodynamic acceptability. Based on considerations of a free energy (or elastic strain energy) potential, [8] derive the following stiffness matrix, which can be used directly in, for instance, a finite element program for general stress states, ensuring fully conservative elastic behaviour when the moduli are functions of pressure,

$$D_{ijkl} = p_a \left(\frac{p_o}{p_a} \right)^{n_h} \left[n_h K_h \frac{\sigma'_{ij} \sigma'_{ikl}}{p_o^2} + K_h (1 - n_h) \delta_{ij} \delta_{kl} + 2G_h \left(\delta_{ik} \delta_{jl} - \frac{1}{3} \delta_{kl} \delta_{ij} \right) \right] \quad (12)$$

where, p_o , is a function of stresses defined in (11); p_a , is the atmospheric pressure taken equal to 100 kPa; K_h , is a dimensionless bulk stiffness factor; G_h , is a dimensionless shear stiffness factor and n_h , is a dimensionless pressure exponent.

$$p_o^2 = \frac{\sigma'_{mm} \sigma'_{nn}}{9} + \frac{K_h (1 - n_h) s_{mn} s_{mn}}{2G_h} \quad (13)$$

The form of the stiffness matrix in equation (10) has two important consequences: i) the moduli depend on all the stress invariants (not just the mean stress) ii) the elastic response is anisotropic.

3.2 Modified plastic hardening

The kinematic hardening embedded in KHSM does not predict a smooth transition from elastic to elasto-plastic behaviour. To do so, [9], the value of hardening modulus should be infinite when the stress state engaged the yield surface. This is because when the stress state is within the yield surface the strains predicted are elastic and the plastic strains are equal to zero. When the stress state touches the yield surface, both elastic and plastic strains are predicted, so in order to have a smooth transition the plastic strains should be initially equal to zero and hence the hardening modulus equal to infinite. Inspection of equation (7) shows that, when the stress state engages the yield surface, a finite value of the hardening modulus is calculated.

As an alternative to (7) for this the formulation proposed in [9] is therefore adopted as an alternative to compute the hardening modulus,

$$H = H_c + \frac{BP_c^3}{\lambda^* - \kappa^*} \left(\frac{b}{b_{\max}^* - b} \right) R^2 \quad (13)$$

Where, H_c and b were defined in equations (6) and (8) respectively. B is a parameter. The value of b_{\max}^* is set equal to the value of b each time the stress state becomes elasto-plastic (i.e., engages the yield surface). Details of the necessary incremental updating procedure are

given in [10], where the numerical implementation of the model in a FE program is also discussed.

3 PARAMETRIC ANALYSIS IN DRAINED TRIAXIAL TESTS

A parametric analysis of the influence of some parameters of the KHSM was done by the simulation of drained compression triaxial test. Here only some results for the influence of structure parameters or elasticity formulation are shown, more details are presented in [10].

Soils parameters are chose to represent the behavior of intact London Clay and the initial stress conditions before shearing are close to the initial state for triaxial tests of at 12 m of depth given by [11]. Some parameters are fixed (Table 1) and the influence of others is explored (Table 2, with base values in bold characters). Fixed parameters correspond with parameters for reconstituted London Clay given by [12]. The base values of Viggiani's elastic parameters $A_g=300$, $n_g=0.87$ and $m_g=0.28$ were selected based on the plasticity index of 50, representative of London Clay [13] Initial stress state is shown in Table 3, the initial position for the reference surface is derived from the preconsolidation pressure (P_0) value (in the model $P_c = P_0/2$) estimated from oedometer tests reported by [11].

Table 1: Fixed parameters for the sensitivity analysis

λ^*	κ^*	M	$\phi_{cs} [^\circ]$	R	B	$\psi^{(a)}$	A
0.097	0.046	0.85	22	0.02	4	7	0.75

a) Only required for the original Plastic modulus (eq. 7)

Table 2: Exploratory parameters for the sensitivity analysis

Parameter	Values explored
k	0, 0.5 , 1, 2
r_0	1, 4 , 8
A_g	100, 300 , 700
n_g	0.5, 0.7, 0.87
m_g	0.2, 0.28 , 0.4
n_h	0.5, 0.75, 0.9
K_h	100, 300, 700
G_h	100, 300, 700

Table 3: Initial stress state for the sensitivity analysis

p' [kPa]	q [kPa]	P_c [kPa]
260	-90	400

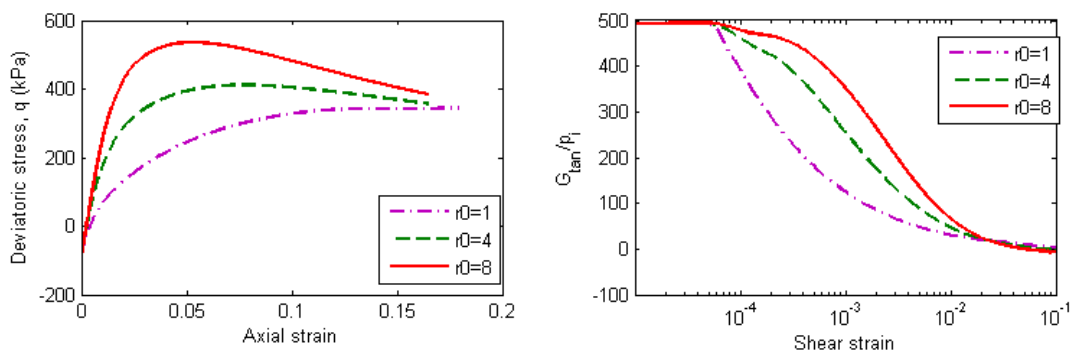
Results of the sensitivity analysis are shown in terms of stress-strain and stiffness-strain curves. The stiffness plotted is the octahedral shear stiffness $3G_{oct} = dq/d\varepsilon_s$, where $q = [3/2(s:s)]^{0.5}$ is the generalized shear stress and $\varepsilon_s = [2/3(e_s:e_s)]^{0.5}$ is the generalized shear

strain; \mathbf{e}_s is the deviatoric component of the strain tensor. The superscript “tan” in the figures is used to denote tangent stiffness. The stiffness is further normalized by p'_i , which is the value of p' at the start of shearing.

3.1 Influence of structure parameters

In all the analyses in this section a comparison is made between results obtained using the original plastic modulus formulation (7) and the modified one (eq.13). The influence of structure parameters: r_0 and k is shown in Figure 2. Figure 2 shows a strong influence of the plastic modulus formulation in the stiffness degradation curves. The original plastic modulus predicts a drop in stiffness, faster when r_0 increases, while the modified plastic modulus results in a smoother stiffness degradation. This behaviour is attributed to the non-smooth elasto-plastic transition when the stress state engaged the yield surface (bubble) as was discussed previously. The variation of the plastic modulus (H) with axial strain for the two formulations is shown in Figure 3. A consistent behavior is observed for the modified plastic modulus. A higher initial structure results in slower the decay of the plastic modulus. Then, for given axial strain, higher values of H are observed when r_0 increases and the generation of plastic strains is reduced, this behaviour is consistent with the structure increasing both stiffness and strength. An opposite effect is observed for the original plastic modulus formulation which is reflected in decreasing stiffness in the small strain region when r_0 increases.

The effect of the rate of destructuration (k) can be seen in Figure 4. In general a higher destructuration rate induces lower strength values. The destructuration effect on stiffness is negligible. When the original plastic modulus is employed (Figure 4b) a marked peak strength appears as the rate of destructuration increases. Also a non-smooth stiffness degradation and a non-physical effect in the small strain region of increase in stiffness as the degradation rate increases.



(a)

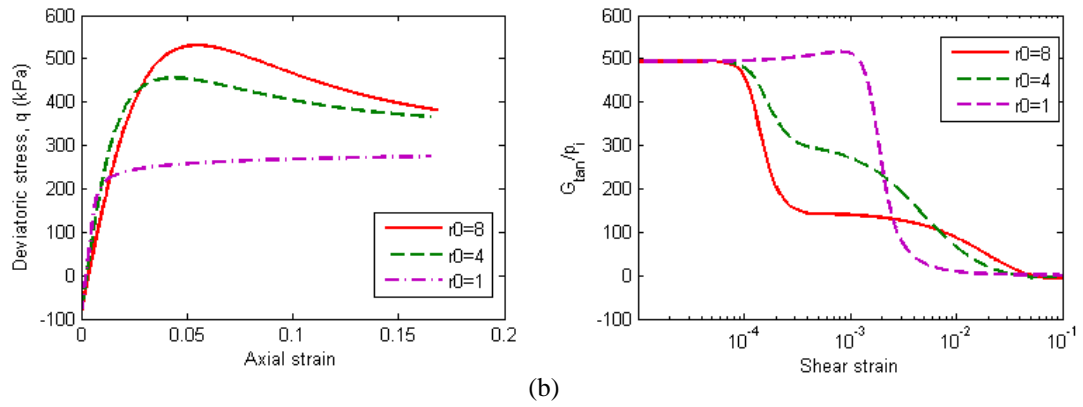


Figure 2: Influence of initial structure. (a) Modified H , (b) Original H

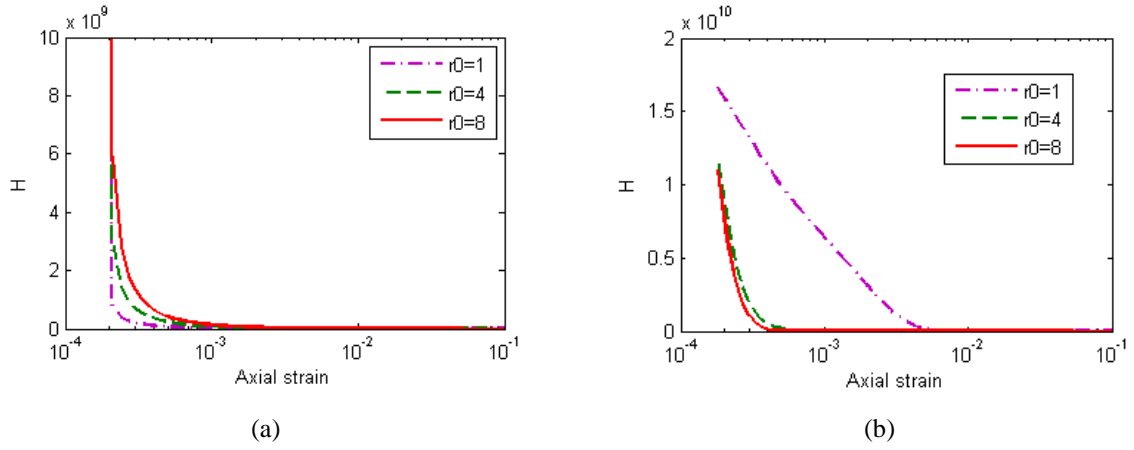


Figure 3: Comparison of plastic modulus (H) evolution. (a) Modified H , (b) Original H

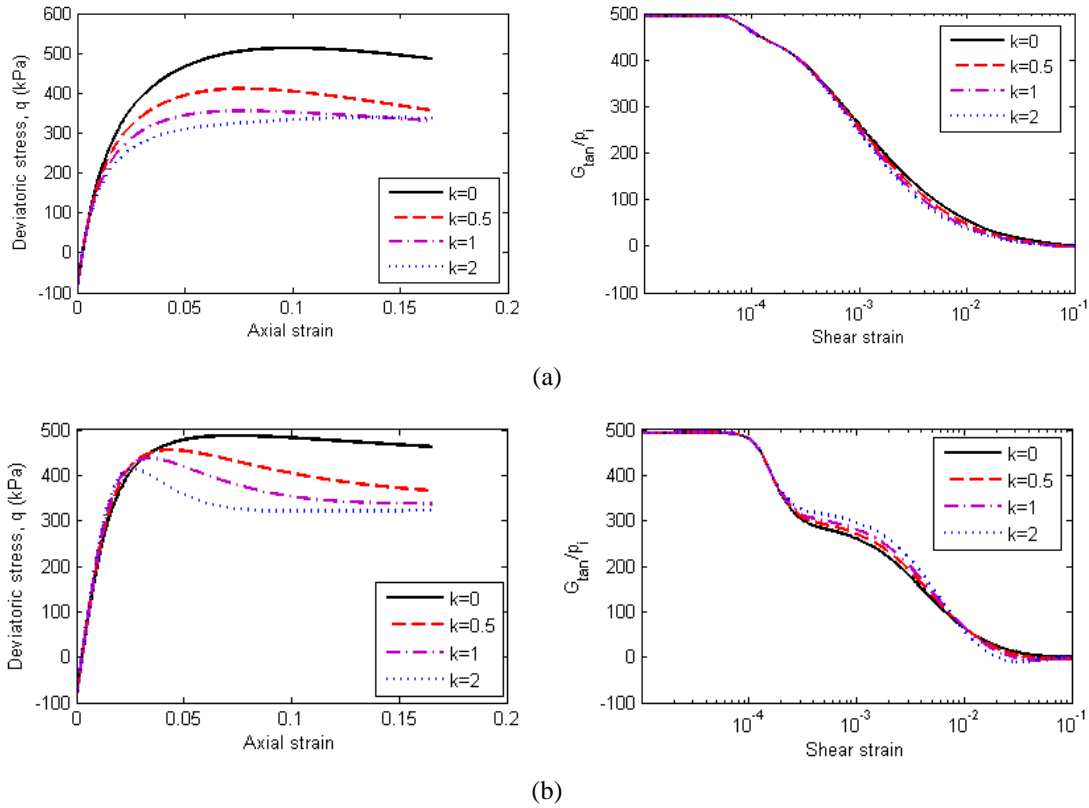


Figure 4: Influence of the rate of destructuration. (a) Modified H , (b) Original H

3.2 Influence of elasticity laws

We examine the influence of the parameters in the different elasticity formulations. Simulations using Viggiani's and Hyper-elasticity are shown in Figure 5 and Figure 6, respectively. The set of constant parameters selected is: $R=0.02$, $B=4$ and $r_0=1$, hence a non-structured case is examined. The modified plastic modulus equation was used for all the analysis in this section. The traditional formulation (10) was also employed, for comparison, using a constant Poisson's ratio of $\nu=0.2$.

Figure 5 shows that parameters A_g and n_g of Viggiani elasticity law have large influence on soil stiffness at very small strains, while the influence of m_g is almost negligible. At large strains ($\gamma > 0.1\%$) n_g still shows an influence on stiffness when $n_g < 0.7$. Following [7] the value of n_g is in the region of 0.5-0.9 depending on the plasticity index. Low values of n_g are applied to soils with low plasticity index which is not the case of London Clay.

For the triaxial compression test, bulk stiffness factor (K_h) of Hyper-elasticity law shows a negligible influence on initial shear stiffness and stiffness degradation curves as is shown in Figure 6(a). Shear stiffness factor (G_h) and n_h have a significant influence on initial stiffness in a similar way to the effects of A_g and n_g , respectively, of Viggiani's elasticity law.

It is also observed in Figures 5 and 6 that traditional elasticity law reaches very low shear stiffness at small strains which is reflected in the shape of stress-strain curve. Only at shear

strains greater than 1% traditional elasticity shows similar stiffness values than the others elasticity laws. Regardless of the elasticity law used the same ultimate stress level is reached at large strains.

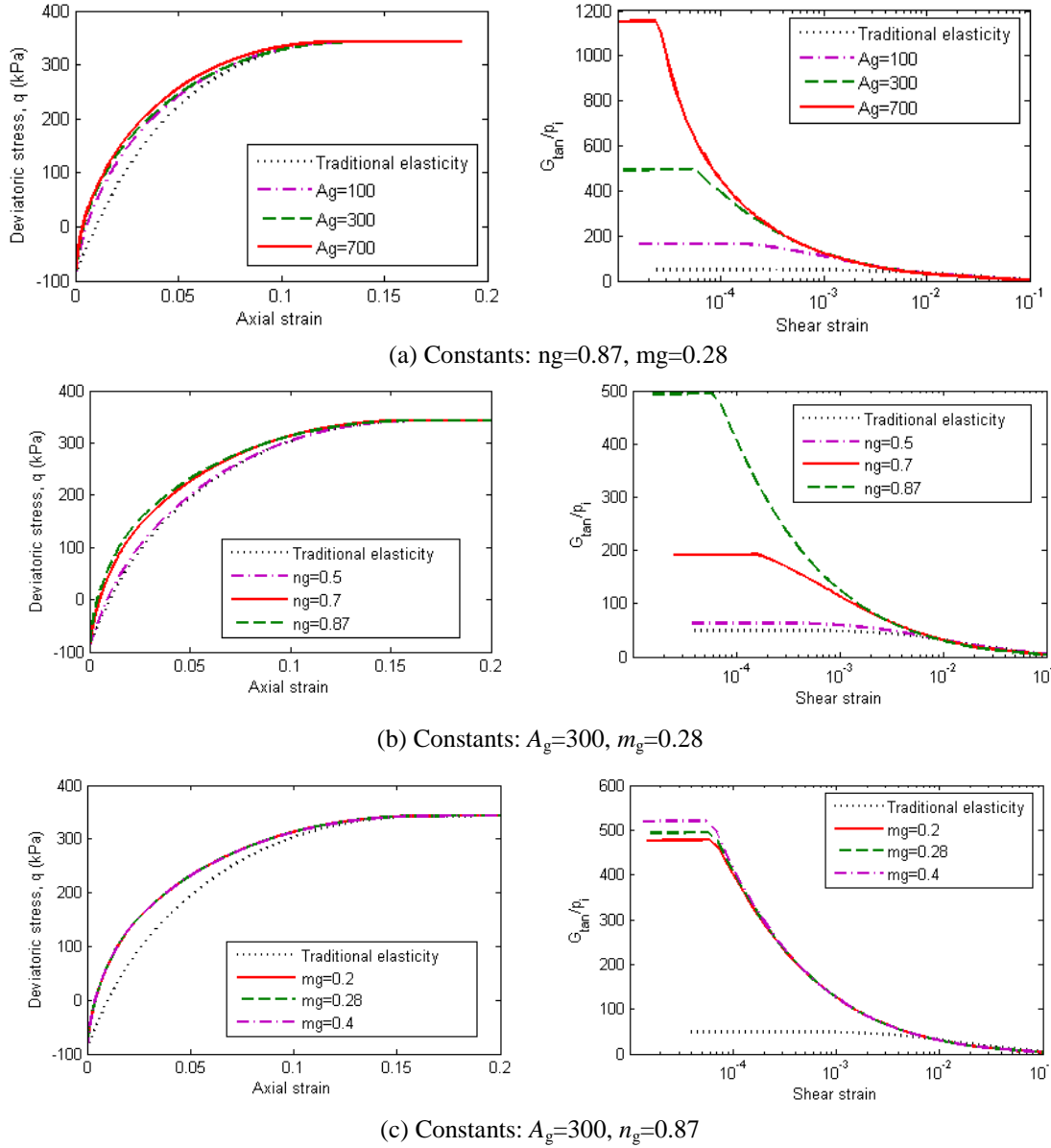
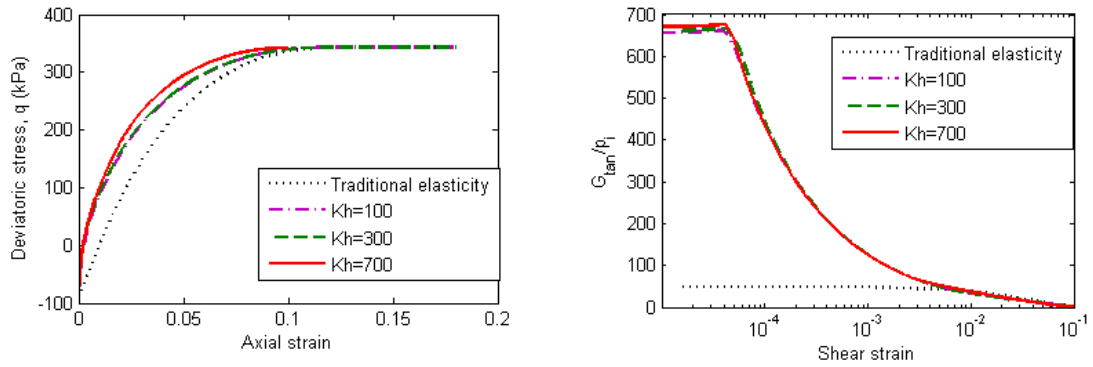
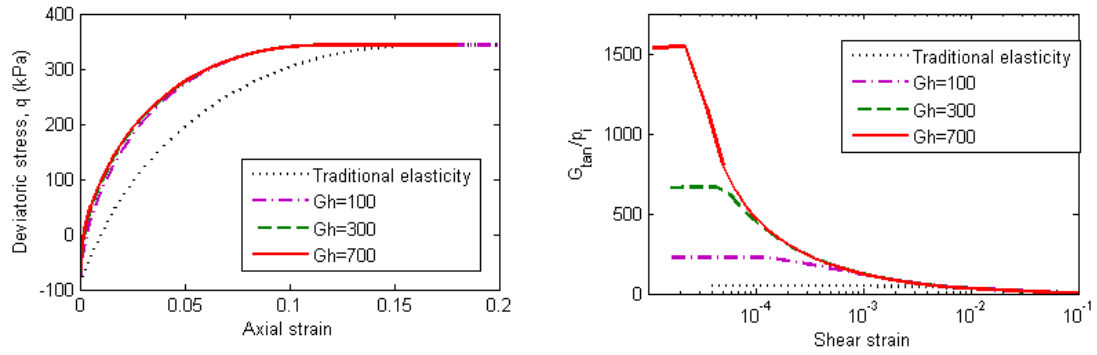


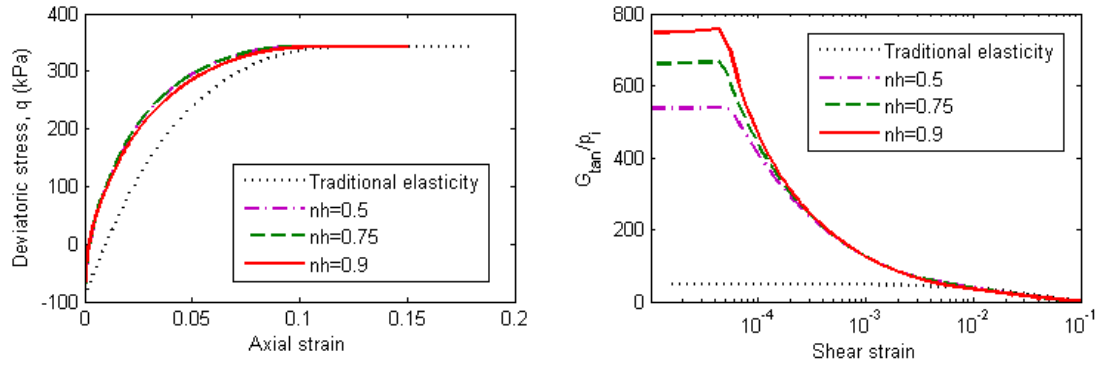
Figure 5: Influence of Viggiani's elasticity law. (a) A_g , (b) n_g , (c) m_g



(a) Constants: $G_h=300$, $n_h=0.75$



(b) Constants: $K_h=300$, $n_h=0.75$



(c) Constants: $K_h=300$, $G_h=300$

Figure 6: Influence of Hyper-elasticity law parameters. (a) K_h , (b) G_h , (c) n_h

4 CONCLUSIONS

Initial structure of the soil is a state variable which modified both strength and stiffness of the soil. If the original plastic modulus is used, an anomalous stiffness degradation will be observed due to the abrupt drop in stiffness observed when structure increases. The structure parameter k shows a negligible influence on stiffness, but not on strength. In case of modified

plastic modulus, k affects both peak strength and residual strength, while only residual strength is affected when original plastic modulus is used. In addition, modified plastic modulus seems to result in a less brittle response than the original one. Two alternative formulations have been used to describe the elastic behaviour of the soil. It has been shown that both formulations give equivalent results for the chosen parameters. In contrast, the traditional elastic formulation does not attain high initial stiffness at small strains.

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